Polarized Chirped Soliton Oscillations in a Lossy Elliptically Low Birefringent Optical Fiber

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Using a variational method, a new model of the nonlinear propagation of optical solitons generated from semiconductor lasers through a lossy elliptically low birefringent fiber, is presented within the framework of a system of coupled nonlinear Schroedinger (CNLS) equations. This model demonstrates polarized soliton oscillations in a lossy elliptically low birefringent fiber.

Key words: Solitons; Birefringence; Polarization; Optical Fiber.

Introduction

It has been recently recognized that optical solitons present a unique opportunity for performing a range of all-optical processing functions in fibers. This observation was based upon the fact that, even in nonlinear fiber systems that have no exact solitons, an injected soliton like pulse displays a degree of phase coherence over the whole pulse, meaning thereby that it would be possible to process an individual soliton bit. These prospects motivate important research efforts towards the development of models [1–7] describing soliton propagation in birefingent optical fibers under different conditions. In this communication, and adiabatic approach has been followed, using a time-averaged variational principle [8] for the first time, to describe polarization dynamics of frequency chirped solitons in a lossy elliptically low birefringent optical fiber.

Model

The soliton propagation in a low birefringent optical fiber is described by the following coupled nonlinear Schroedinger (CNLS) equations [1, 4]:

$$\begin{aligned} &i(u_z+\delta u_t)+\frac{1}{2}u_{tt}+(|u|^2+B|v|^2)\,u+A\,v^2\,u^*\\ &\cdot\exp(-iR\delta z)+Du^2\,v^*\exp\left(\frac{1}{2}iR\delta z\right)\\ &+D(2|u|^2+|v|^2)\,v\exp\left(-\frac{1}{2}iR\delta z\right)=-i\gamma u, \end{aligned} \tag{1a}$$

$$\begin{aligned} &i(v_z-\delta v_t)+\frac{1}{2}v_{tt}+(B|u|^2+|v|^2)\,v+A\,u^2\,v^*\\ &\cdot\exp(iR\delta z)+Dv^2\,u^*\exp\left(-\frac{1}{2}iR\delta z\right)\\ &+D(|u|^2+2|v|^2)\,u\exp\left(\frac{1}{2}iR\delta z\right)=-i\gamma v, \end{aligned} \tag{1b}$$

where $B=\dfrac{2(1+\varepsilon\sin^2\theta)}{2+\varepsilon\cos^2\theta}$, $A=\dfrac{\varepsilon\cos^2\theta}{2+\varepsilon\cos^2\theta}$, $D=\dfrac{\varepsilon}{2}\tan\theta$

with $\theta=35^0$.

u and v are the normalized amplitudes of the fast and slow modes, respectively, t is the time coordinate in a frame moving with the average group velocity of the two modes measured in units of the modulational wavelength, z is the distance along the propagation direction, δ is the normalized birefringence, ε denotes the relative strength of the cross-phase modulation, R is the wave vector mismatch due to modal birefringence of the fiber, and γ denotes the losses in the fiber. The oscillating or coherence terms appearing in (1) arise from nonlinear polarization and are responsible for the polarization instability in low birefringence fibers in which the slowly moving partial pulse transfers energy to the fast moving partial pulse. In the case of a single nonlinear Schroedinger equation, modulational instability gain occurs only in the anomalous group-velocity dispersion regime. However, when birefringence is added to the phase matching condition there can also be cross-modulational instability in the normal dispersion regime. Modulational instability can be considered as a parametric four-wave mixing process in

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which the nonlinearity explicitly enters the phase-matching condition. Using the following transformation [5]

$$p = \sqrt{\frac{B}{2}}\,(u e^{i\alpha z} + v e^{-i\alpha z}) \text{ and}$$
$$q = \sqrt{\frac{B}{2}}\,(u e^{i\alpha z} - v e^{-i\alpha z}),$$

the CNLS equations (1) can be written as

$$i(p_z + \delta q_t) + \frac{1}{2} p_{tt} + \alpha q + p(f|p|^2 + h|q|^2) + g p^* q^2 + i\gamma p = 0, \quad (2a)$$

$$i(q_z + \delta p_t) + \frac{1}{2} q_{tt} + \alpha p + q(f'|q|^2 + h|p|^2) + g q^* p^2 + i\gamma q = 0, \quad (2b)$$

where

$$\alpha = \frac{1}{4} R\delta,\ f = \frac{1}{2B}(1 + A + B + 4D),$$
$$f' = \frac{1}{2B}(1 + A + B - 4D),\ h = \frac{1}{B}(1 - A),$$
$$\text{and } g = \frac{1}{2B}(1 + A - B).$$

Since losses in the fiber lead to exponential decrease of soliton amplitude, one can use the transformation $p \to p'e^{-\gamma z}$ and $q \to q'e^{-\gamma z}$ and write (2) in the form

$$i(p'_z + \delta q'_t) + \alpha q' + \frac{1}{2} p'_{tt} + p'(f|p'|^2 + h|q'|^2) \cdot e^{-2\gamma z} + g p'^* q'^2 e^{-2\gamma z} = 0, \quad (3a)$$

$$i(q'_z + \delta p'_t) + \alpha p' + \frac{1}{2} q'_{tt} + q'(f'|q'|^2 + h|p'|^2) \cdot e^{-2\gamma z} + g q'^* p'^2 e^{-2\gamma z} = 0. \quad (3b)$$

The spatiotemporal evolution of the wave amplitudes is governed by the Lagrangian density

$$L = \int_{-\infty}^{\infty} \Bigg[\frac{i}{2}(p'_z p'^* - p'^* z p') + \frac{i}{2}(q'_z q'^* - q'^*_z q') + \frac{i\delta}{2}(p'_t q'^* - q'^*_t p') + \frac{i\delta}{2}(q'_t p'^* - p'^*_t q') + \alpha(p'^* q' + p' q'^*) + h|p'|^2|q'|^2 e^{-2\gamma z} + \frac{1}{2}(f|p'|^4 e^{-2\gamma z} - |p'_t|^2) + \frac{1}{2}(f'|q'|^4 e^{-2\gamma z} - |q'_t|^2) + \frac{1}{2} g(p'^2 q'^{*2} + q'^2 p'^{*2}) e^{-2\gamma z} \Bigg] \mathrm{d}t, \quad (4)$$

where $p'_z = \dfrac{\partial p'}{\partial z}$, $p'_t = \dfrac{\partial p'}{\partial z}$, and so on.

The soliton solutions of (3), using a variational method, are based upon the trial function [5]

$$\begin{bmatrix} p' \\ q' \end{bmatrix} = 2\eta_r \exp[2i V_r (t - \zeta_r) - iC \tanh[2\eta_r (t - \zeta_r)](t - \zeta_r)^2 + i D_r] \operatorname{sech}[2\eta_r (t - \zeta_r)], \quad (5)$$

describing the temporal form of the pulses. The evolution parameters η_r, ζ_r, V_r and D_r ($r = 1, 2$ correspond to p' and q' solitons, respectively), and C represent the amplitude, central position, velocity of solution's central position as it propagates along the fiber, phase, and initial frequency chirp of the solution, respectively. Substitution of (5) into the Lagrangian density (4) and using Euler-Lagrange equations give rise to the following system of coupled ordinary differential equations (ODEs) for the evolution of soliton parameters:

$$\frac{\mathrm{d}}{\mathrm{d}z}(\eta_r V_r) = \frac{1}{8}\frac{\partial L_1}{\partial \zeta_r} + \frac{1}{8}\frac{\partial L_2}{\partial \zeta_r} + \frac{1}{8}\frac{\partial L_3}{\partial \zeta_r}, \quad (6)$$

$$8\eta_r \frac{\mathrm{d}\zeta_r}{\mathrm{d}z} = 16\eta_r V_r - 2C\alpha_1 - 4C - \frac{\partial L_1}{\partial V_r} - \frac{\partial L_3}{\partial V_r}, \quad (7)$$

$$4\frac{\mathrm{d}\eta_r}{\mathrm{d}z} = (-1)^r \frac{\partial L_1}{\partial \Lambda} + (-1)^r \frac{\partial L_3}{\partial \Lambda}, \quad (8)$$

$$\frac{\mathrm{d}D_1}{\mathrm{d}z} = 2V_1 \frac{\mathrm{d}\zeta_1}{\mathrm{d}z} + 4f\eta_1^2 e^{-2\gamma z} - 2V_1^2 - 2\eta_1^2 + \frac{C^2}{16\eta_1^2}\alpha_2 + \frac{C^2}{4\eta_1^2}\alpha_4 + \frac{C^2}{4\eta_1^2}\alpha_5 + \frac{1}{4}\frac{\partial L_1}{\partial \eta_1} + \frac{1}{4}\frac{\partial L_2}{\partial \eta_1} + \frac{1}{4}\frac{\partial L_3}{\partial \eta_1}, \quad (9a)$$

$$\frac{\mathrm{d}D_2}{\mathrm{d}z} = 2V_2 \frac{\mathrm{d}\zeta_2}{\mathrm{d}z} + 4f'\eta_2^2 e^{-2\gamma z} - 2V_2^2 - 2\eta_2^2 + \frac{C^2}{16\eta_2^2}\alpha_2 + \frac{C^2}{4\eta_2^2}\alpha_4 + \frac{C^2}{4\eta_2^2}\alpha_5 + \frac{1}{4}\frac{\partial L_1}{\partial \eta_2} + \frac{1}{4}\frac{\partial L_2}{\partial \eta_2} + \frac{1}{4}\frac{\partial L_3}{\partial \eta_2}, \quad (9b)$$

where

$$\alpha_1 = \left(\frac{\pi^2}{9} - \frac{2}{3}\right),\quad \alpha_2 = \left(\frac{7\pi^4}{225} - \frac{\pi^2}{3} + \frac{2}{5}\right),$$
$$\alpha_3 = 1,\quad \alpha_4 = \left(\frac{\pi^2}{18} + \frac{2}{3}\right),\quad \alpha_5 = \left(\frac{\pi^2}{12} - \frac{1}{2}\right),$$

$$L_1 = 4\alpha\eta_2 \int_{-\infty}^{\infty} \operatorname{sech} x_1 \operatorname{sech} x_2 \cos \Lambda \, \mathrm{d}x_1 = \frac{8\alpha\eta\varrho\cos\Lambda}{\sinh\varrho},$$

$$L_2 = 8h\eta_1\eta_2^2 e^{-2\gamma z} \int_{-\infty}^{\infty} \operatorname{sech}^2 x_1 \operatorname{sech}^2(x_1+\varrho)\, \mathrm{d}x_1 = 32h\eta^3 e^{-2\gamma z} \frac{\cosh\varrho}{\sinh\varrho}(\varrho - \tanh\varrho),$$

$$L_3 = 8g\eta_1\eta_2^2 e^{-2\gamma z} \cdot \int_{-\infty}^{\infty} \operatorname{sech}^2 x_1 \operatorname{sech}^2(x_1+\varrho)\cdot\cos 2\Lambda\, \mathrm{d}x_1 = 32\, g\eta^3 e^{-2\gamma z}\cos\Lambda \frac{\cosh\varrho}{\sinh^3\varrho}(\varrho - \tanh\varrho),$$

and

$$\Lambda = 2t(V_1 - V_2) - 2(V_1\zeta_2 - V_2\zeta_2) - (D_2 - D_1) - C\tanh[2\eta_1(t-\zeta_1)](t-\zeta_1)^2 + C\tanh[2\eta_2(t-\zeta_2)](t-\zeta_2)^2.$$

The above integrals are evaluated for nearly equal pulse amplitudes $\eta_1 \cong \eta_2 \cong \eta$, relative phase $\phi = D_2 - D_1$, relative between two polarization maxima $\varrho = x_2 - x_1$, and for $V_1 \cong V_2 \cong V$. The relative parameters, η_{12}, V_{12}, ϕ, and ϱ defined for p' and q' solitons are obtained as follows:

Writing $\eta_{12} = \eta_1 - \eta_2$ and using (8), one gets

$$\frac{\mathrm{d}\eta_{12}}{\mathrm{d}z} = 4\alpha\eta\left(\frac{\varrho}{\sinh\varrho}\right)\sin\left(\frac{V}{\eta}\varrho+\phi\right) - 32g\eta^3 e^{-2\gamma z}\left[\frac{\cosh\varrho}{\sinh^3\varrho}(\varrho - \tanh\varrho)\right] \cdot \sin 2\left(\frac{V}{\eta}\varrho+\phi\right). \tag{10}$$

Similarly, writing $V_{12} = V_1 - V_2$ and using (6), one gets

$$\frac{\mathrm{d}V_{12}}{\mathrm{d}z} = -\frac{4}{3}\alpha\eta\varrho\cos\left(\frac{V}{\eta}\varrho+\phi\right) - \frac{64}{15}h\eta^3\varrho e^{-2\gamma z} - \frac{64}{15}g\eta^3\varrho e^{-2\gamma z} \cdot \cos 2\left(\frac{V}{\eta}\varrho+\phi\right). \tag{11}$$

Also, from $\varrho = 2\eta(\zeta_1 - \zeta_2)$ and using (7), (10), and (11), one gets

$$\frac{\mathrm{d}^2\varrho}{\mathrm{d}z^2} = -\frac{16}{3}\alpha\eta^2\varrho\cos\left(\frac{V}{\eta}\varrho+\phi\right) - \frac{256}{15}h\eta^4\varrho e^{-2\gamma z} - \frac{256}{15}g\eta^4\varrho e^{-2\gamma z}\cos 2\left(\frac{V}{\eta}\varrho+\phi\right) - 2\left(\frac{\pi^2}{9}+\frac{4}{3}\right)C\alpha\sin\left(\frac{V}{\eta}\varrho+\phi\right) - \frac{16}{3}\left(\frac{\pi^2}{9}+\frac{4}{3}\right)Cg\eta^2 e^{-2\gamma z}\sin 2\left(\frac{V}{\eta}\varrho+\phi\right). \tag{12}$$

The two solitons with opposite phases form a bound state provided

$$\frac{V}{\eta}\varrho + \phi = \Phi = \pm\pi.$$

Thus, one can write (12) as

$$\frac{\mathrm{d}^2\varrho}{\mathrm{d}z^2} = \frac{16}{3}\alpha\eta^2\varrho - \frac{256}{15}h\eta^4\varrho e^{-2\gamma z} - \frac{256}{15}g\eta^4\varrho e^{-2\gamma z}$$

or

$$\frac{\mathrm{d}^2\varrho}{\mathrm{d}z^2} + (a e^{-2\gamma z} - b)\varrho(z) = 0, \tag{13}$$

where $a = \frac{256}{15}\eta^4(h+g)$ and $b = \frac{16}{3}\alpha\eta^2$.

Equation (13) can also be written as

$$\sigma^2\frac{\mathrm{d}^2\varrho}{\mathrm{d}\sigma^2} + \sigma\frac{\mathrm{d}\varrho}{\mathrm{d}\sigma} + \left(\sigma^2 - \frac{b}{\gamma^2}\right)\varrho = 0, \tag{14}$$

where $\sigma = \frac{\sqrt{a}}{\gamma}e^{-\gamma z}$.

Equation (14) is a well known Bessel Equation. Its general solution is given by

$$\varrho(\sigma) = X J_{\frac{\sqrt{b}}{\gamma}}(\sigma) + Y J_{-\frac{\sqrt{b}}{\gamma}}(\sigma). \tag{15}$$

Considering γ to be small, one can use an asymptotic expansion of the Bessel function to write (15) as

$$\varrho(\sigma) \cong \left(\frac{2\gamma}{\pi\sqrt{a}\,e^{-\gamma z}}\right)^{\frac{1}{2}} \cdot \cos\left(\frac{\sqrt{a}}{\gamma}(1-\gamma z + o(z^2)) - \frac{\pi}{2}\frac{\sqrt{b}}{\gamma} - \frac{\pi}{4}\right). \tag{16}$$

The frequency ω of relative oscillations of soliton positions is given by

$$\omega^2 = a = \frac{256}{15}\eta^4\left(\frac{2}{B} - 1\right). \tag{17}$$

The oscillatory behavior of solitons in nonintegrable dynamical systems leads to the radiation of a small amplitude wave by the solitons during their propagation through the lossy low birefringent fiber. The intensity of radiation will be proportional to the amplitude of the oscillations. The energy of the soliton will, therefore, decrease as it propagates in a lossy elliptically low birefringent optical fiber.

Conclusions

Admittedly, this model of chirped soliton dynamics demonstrating polarized soliton dynamics in a lossy low birefringent fiber is simplified in order to get some physical insight. Then, by insisting on preserving simplicity at the expense of quantitative accuracy, the frequency of relative oscillations of soliton positions has been obtained, taking into account the interaction between different polarizations in a lossy elliptically low birefringent fiber. If more precision is desired, then one has to include radiation effects into this analytical formulation. This may be particularly important when long propagation distances are considered.

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